A MULTI-COLONY ANT SYSTEM FOR COMBINATORIAL OPTIMIZATION PROBLEM

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A multi-colony ant system (MAS) is proposed for the combinatorial optimization problems. The proposed MAS is inspired by the knowledge that there are many colonies of ants in the natural world and organized with multiple colonies of ants. At first, ants perform solution search procedure by cooperating with each other in the same colony until no better solution is found after a certain time period. Then, communication between different colonies is performed to build new pheromone distributions for each colony, and ants start their search procedure again in each separate colony, based on the new pheromone distribution. The proposed algorithm is tested by simulating the traveling salesman problem (TSP). Simulation results show that the proposed method performs better than the traditional ACO.

Keywords: Evolutionary algorithm; ant colony optimization; multi-colony; combinatorial optimization problems; traveling salesman problem.

1. Introduction

Ant colony optimization (ACO) is a recently developed, population-based approach which has been successfully applied to a number of combinatorial optimization problems[1] arising in many different fields such as economy, commerce, engineering, industry or medicine.[2–5] The inspiring source of ACO is the foraging behaviors of real ants. In the natural world, when ants attempt to find short paths between their nest and food sources, they communicate among the individuals of their colony indirectly by using a kind of chemical substance, called pheromone. As the similar searching fashion used by ants in the real world, artificial ants in ACO search for solutions in an iterative fashion, that is, ants of the next iteration build solutions on the basis of the new pheromone information, which is updated by ants of the last iteration after finishing their solution constructions. By exploiting this mechanism, ACO has been applied to solve a number of combinatorial optimization problems, for example, the traveling salesman problem,[6–9] the quadratic assignment problem,[10] the scheduling problem,[11] the vehicle routing problem[12] and so on.

Especially, since the first ACO was proposed, it has been proved to be an effective algorithm to solve the TSP problem, and many extensions of ACO were introduced. Since the search behavior of ACO can be characterized by two main features,[13–15] which are intensification and diversification, and the balance between them greatly affects the performance of ACO, in the extensions of ACO, many efforts were done to balance the intensification and diversification. However, most of means of balancing them refer either to pheromone updating, or to combination with other algorithms. In this research, we proposed a variant of ACO algorithm called multi-colony ant system. As the name shown, the proposed method exploits multiple colonies to balance the intensification and diversification, while the traditional ones mainly use a single colony. As a merit, the proposed method enhances the diversification by using various information of multi-colonies, and maintains the intensification by retaining the same pheromone updating method as the traditional ACO in each population.

In the remainder, we describe the traditional ACO and the proposed ACO in detail in Secs. 2 and 3 respectively. Then, Sec. 4 demonstrates the effectiveness by some simulation results. Finally, a conclusion is given in Sec. 5.

2. Ant Colony Optimization

In the proposed algorithm, besides the multi-colony mechanism, the pheromone updating method of the traditional ACO is also applied. In this section, we give a brief overview of the traditional ACO algorithm.

Since the first ACO algorithm, called Ant System (AS) was proposed by Dorigo in 1992,[16] the ACO algorithm attracted the attention of more researchers and a number of extensions were introduced and applied to the traveling salesman problem (TSP). In TSP, a given set of n cities has to be traversed so that every city is visited exactly once and the tour ends in the initial city. The optimization goal is to find a shortest possible tour.

The first improvement over AS was obtained by the Elitist AS (EAS),[17] which has a modified pheromone update rule, that is, each time the pheromone trails are updated, those belonging to the edges of the global best tour get an additional amount of pheromone so as to emphasize information on the best solution in the search procedure. The biggest problem that can be caused by such an exploitative method is insufficient exploration and premature convergence to sub-optimal solutions. In Rank-based version of AS (Rank-based AS),[18] the concept of ranking was applied and extended as follows. A number of best ants of the current iteration are allowed to deposit pheromone according to their ranks, while the best-so-far ants are allowed to deposit pheromone of the highest quantity. Another improvement over Ant System is Max–Min Ant System (MMAS),[8,19,20] of which the characterizing elements are that it also adopts a concept of elitism in which only the best ant at each iteration updates trails and that the possible trail values are restricted to the interval $[\tau_{\min}, \tau_{\max}]$, where the two parameters are set up in a problem-dependent way. Ant Colony System (ACS) [21,22] introduced by Dorigo uses a more aggressive action choice rule than AS, called pseudo random proportional rule which favors transitions towards nodes connected by short edges and with a large amount of pheromone. In ACS the pheromone updating rule is only applied to the edges belonging to the global-best tour, and a local pheromone updating rule, by which each time an ant uses an edge (i,j) to move from city i to city j, it removes some pheromone from the edge, is also adopted.

Although modifications on preventing ant from converging to local optimum through exploiting new pheromone update rules have been carried out and acquired relatively good performance, balancing the intensification and diversification is still the most important theme in the study of ACO algorithms. In this research, instead of using one single colony to search for good solutions, we introduced multi-colonies and in each colony, the traditional ACO is implemented separately. In order to share information among the colonies, periodical communication among colonies is also introduced.

3. Multi-colony Ant System

As the traditional ACO was motivated by the foraging behavior of ants in the real world, we developed the multi-colony mechanism based on the fact that there are many colonies in the natural world.

The proposed method consists of two main procedures. In the first procedure, as we assume that artificial ants can only smell pheromone trails among their colony, ants perform solution search procedure by cooperating with each others in the same colony using the traditional rule of ACO, until no better solution is found after a certain time period. In the second procedure, ants in each colony begin to communicate among colonies to prepare new pheromone distribution for the next searching procedure. In order to describe the proposed algorithm in more detail, we first give the framework of it as Fig. 1.

3.1. *Solution search by ants*

From Fig. 1 we know that pheromone information in each colony is initialized first, and then in each separate colony, the solution construction and pheromone update is implemented iteratively by the rule of traditional ACO. In the proposed method, we applied ACS[9,22] as the traditional ACO. Next, we give a simple introduction of ACS by some equations on the basis of mathematical model of TSP.

3.2. *Framework of multi-colony ant system*

In TSP, for a set of cities, we consider d_{ij} to be the distance between any given cities i and j, such that the path length $d_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{\frac{1}{2}}$, where (x_i, y_i), (x_j, y_j) is the coordinates of city i and city j. Let the τ_{ij} be the amount of pheromone in the edge that connects i and j. Initially, each of m ants is put on some randomly chosen city, and then decides independently which city to go to using a transition rule that is

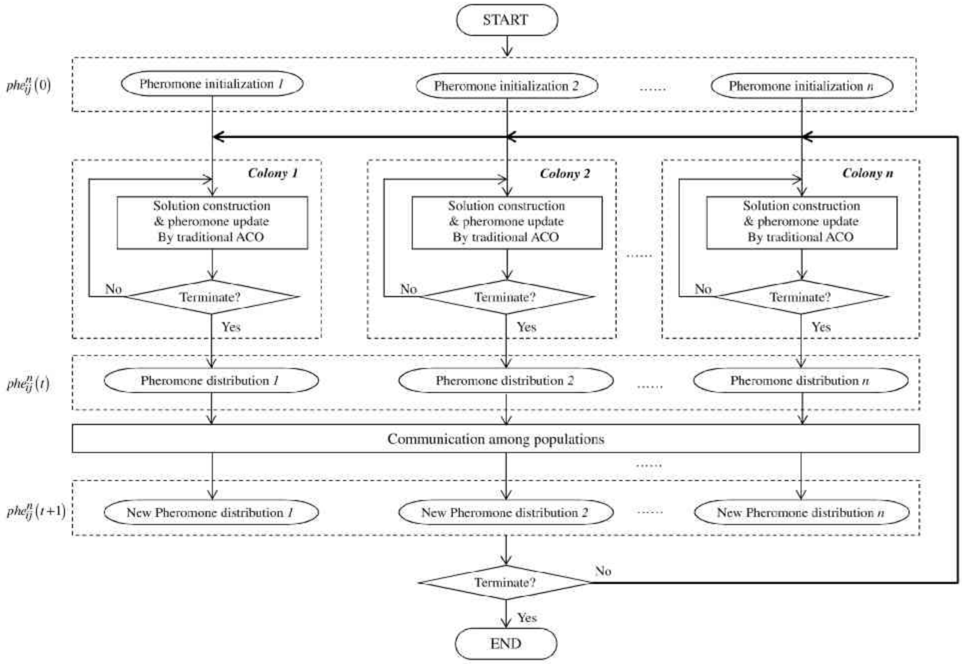

Fig. 1. The framework of multi-colony ant system.

a function of the distance to the city and the amount of pheromone of the present connecting paths, until the tour is completed. The probability, which shows the transition rule of the kth ant making the transition at iteration t from city i to city j is given by:

$$p_{ij}^k(t) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}(t) \cdot \eta_{ij}^{\beta}}{\sum_{l \in J_i^k} \tau_{il}^{\alpha}(t) \cdot \eta_{il}^{\beta}} & \text{if } j \in J_i^k(t) \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $\eta_{ij} = 1/d_{ij}$ is a heuristic value, α and β are two parameters which determines the relative importance of pheromone value and heuristic information, and $J_i^k(t)$ is the feasible neighborhood of ant k at iteration t, that is, the set of cities remain to be visited by ant k positioned at city i. After all ants have built a tour, ants perform following pheromone update rule:

$$\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \rho \cdot \Delta\tau_{ij}^{bs}(t), \forall (i,j) \in T^{bs}, \tag{2}$$

where $\rho \in (0,1)$ is the evaporation rate of the pheromone trail, and $\Delta\tau_{ij}^{bs} = 1/C^{bs}$. It is important to note that in ACS the pheromone trail update, both evaporation and new pheromone deposit, only applies to the arcs of T^{bs}, not to all the arcs as in AS.

In addition to the global pheromone trail updating rule shown above, in ACS the ants use a local pheromone update rule that they apply immediately after having crossed an arc (i, j) during the tour construction:

$$\tau_{ij} \leftarrow (1-\xi) \cdot \tau_{ij}(t) + \xi \cdot \tau_0, \tag{3}$$

where $\xi(0 < \xi < 1)$, and τ_0 are two parameters. The value of τ_0 is set to be the same as the initial value for the pheromone trails.

3.3. *Communication among colonies*

After the search procedure is separately implemented in each colony, the pheromone of each colony formed new distribution. In the proposed method, in order to enhance the diversification, the pheromone communication is done among the colonies. In each colony, the pheromone distribution reaches to a relative stable state after meeting the termination conditions. Through pheromone communication, the relative stable state turns to be relatively unstable, and this state contribute to the next solution search procedure through enhancing diversification.

As to the means of the communication, we first assume that n colonies are applied, and the pheromone distribution in each colony is presented by $\mathrm{phe}_{ij}^{n}(t)$, which represents the pheromone concentration of $\mathrm{arc}(i, j)$ in colony n after t time of communication. Note that $\mathrm{phe}_{ij}^{n}(0)$ is the initialized pheromone distribution. Before each communication, pheromone distribution is acquired as $\mathrm{phe}_{ij}^{n}(t)$ and we know that there are totally n colonies as follows:

$$\mathrm{phe}_{ij}^{0}(t), \mathrm{phe}_{ij}^{1}(t), \ldots, \mathrm{phe}_{ij}^{n}(t). \tag{4}$$

In order to acquire new pheromone distributions $\mathrm{phe}_{ij}^{n}(t+1)$, an important procedure in the proposed method is applied. That is randomly selecting two colonies and averaging their concentrations as follows:

$$\mathrm{phe}_{ij}^{n}(t+1) = \frac{\mathrm{phe}_{ij}^{r1}(t) + \mathrm{phe}_{ij}^{r2}(t)}{2}, \tag{5}$$

where $r1, r2$ are two random numbers from $[1, n]$. So $2n$ times of randomly selecting are needed. Through this means of communication, new pheromone distributions $\mathrm{phe}_{ij}^{n}(t+1)$ are prepared for the next search procedure, and based on the new pheromone distributions, ants start their search procedure in each separate colony. In order to make the flow of the proposed method more clear, we introduced the outline of outline of MAS in pseudo-code as Fig. 2.

4. Simulation

In this section, we compare the performance of the proposed improvements over AS using some symmetric TSP instances which had been proposed for the First International Contest on Evolutionary Optimization.[22] In order to assess the effectiveness of the proposed ACO algorithm, extensive simulations were carried out over TSPLIB benchmark problems on a PC station (Intel, 2.66GHz). As the parameter setting of

```
algorithm Multi-colony Ant System
  set parameters
  for i=1 to n do
    initialize pheromone distribution i
  end-for
  while(termination condition not met) do
    for i=1 to n do
      while(termination condition not met) do
        construct ant solutions
        update pheromones by traditional ACO
      end-while
    end-for
    for i=1 to n do
      r1,r2←random numbers from [1,n]
      phe_ij^i(t+1) ← [phe_ij^r1(t) + phe_ij^r2(t)]/2
    end-for
  end-while
```

Fig. 2. The outline of MAS in pseudo-code.

the proposed method, if α is too high compared to β, the ACO algorithm tends to enter stagnation behavior without finding good solutions. If α is too low, the algorithm operates like a repeated construction heuristic and generates good solutions, but cannot exploit the positive feedback. The same is true for ρ. If ρ is too close to zero, most of the global information contained in the trails evaporates immediately and learning does not take place. If ρ is too close to one, there is the danger of early convergence of the algorithm. The setting $\alpha = 1$, $\beta = 5$, $\rho = 0.5$ is suggested to be advantageous in Ref. 13. In addition, through extensive experiments on the proposed method, we found that if the colony size is too small or too big, the proposed method shows bad performance, so the colony size is set to 30.

The comparison is done based on the same calculation time. In this research, we compared the results of three TSP problems on three AS-based algorithm (MMAS, ACS, ASrank) and the proposed algorithm. Performance of each algorithm was compared using $Best_{\mathrm{avg}}$ (average of the best tour length) and $Error$ (average excess rate from optimum length) over 25 runs. Note that we give each run an equal calculation time as traditional ACO algorithms cost.

From Table 1, we note that compared with these traditional algorithms, the average of the best tour length in the proposed method shows the smallest value and has the smallest average excess rate from the optimal length. Because simulations of these compared algorithms (MMAS, ACS and ASrank)[8,19,21] on TSP were executed on the same PC and within the equal calculation time, this result shows that the proposed method can converge to a better result within the same calculation time. As a difference to the traditional algorithms, the multi-colony mechanism applies multiple pheromone distributions to balance intensification and exploitation by periodical communication among colonies. Observing the results, we can conclude

Table 1. Simulation result.

TSP		eil51	kroA100	d198
Optimum		426	21282	15780
MMAS	$Best_{\text{avg}}$	427.1	21291.6	15956.8
	Error	0.258%	0.045%	1.120%
ACS	$Best_{\text{avg}}$	428.1	21420	16054
	Error	0.493%	0.648%	1.736%
ASrank	$Best_{\text{avg}}$	428.8	21394.9	16025.2
	Error	0.657%	0.530%	1.554%
Proposed	$Best_{\text{avg}}$	426.1	21282.9	15932.4
	Error	0.023%	0.004%	0.965%

that the multi-colony mechanism plays an important role to enhance the performance of the proposed method.

5. Conclusion

Multi-colony ant system, as an improved ACO algorithm, has been proposed in this research. In the proposed method, we introduced a multi-colony mechanism. Ants in each colony first search solutions separately by the rule of the traditional ACO, and after the search procedure in each colony terminated, communication among colonies is implemented to prepare new pheromone information for the next search procedure, as a method of enhancing the diversification of the algorithm. The proposed algorithm was applied to the TSP, and to verify the performance of it, several TSP benchmark problems were simulated. From the simulation results, we find that the improved ACO algorithm has very high performance in searching solution compared with other algorithms.

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